A COMMON FIXED POINT THEOREM FOR FOUR SELFMAPS OF A COMPACT *D**−METRIC SPACE

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ABSTRACT

The purpose of this paper is to prove a common fixed point theorem for four selfmaps on a D–metric space and deduce a common fixed point theorem for four selfmaps on a compact D*–metric space. Further we show that a common fixed point theorem for four selfmaps of a metric space prove by Brian Fisher ([5]) is a particular case of our theorem.*

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***Key Words**: D*-metric space, Compatible, Fixed point theorem.*

1. INTRODUCTION AND PRELIMINARIES

Fixed point theory is a fundamental area in nonlinear functional analysis, offering powerful tools for solving a wide range of problems across mathematics and applied sciences. The pioneering work of Banach on contraction mappings laid the foundation for this field. His celebrated fixed point theorem guarantees the existence and uniqueness of fixed points for self mappings in complete metric spaces and ensures convergence via Picard iteration. Over the decades, this classical result has inspired numerous generalizations to broader classes of mappings and more generalized spaces.

Different mathematicians tried to generalize the usual notion of metric space (X, d). In 1992 Dhage [2] has initiated the study of generalized metric space called D- metric space and fixed point theorems for selfmaps of such spaces. Later researchers have made a significant contribution to fixed point of D- metric spaces in [1], [3], and [4]. Unfortunately almost all the fixed point theorems proved on D-metric spaces are not valid in view of papers [6], [7] and [8].

Recently Shaban Sedghi, Nabi Shobe and Haiyun Zhou [9], have introduced D*- metric spaces as a probable modification of D- metric spaces and proved some fixed point theorems.

Definition 1.1([9]): Let X be a non-empty set. A function $D^*: X^3 \to [0, \infty)$ is said to be a **generalized metric** or **D*-metric** or **G-metric** on X, if it satisfies the following conditions

(i) $D^*(x, y, z) \geq 0$ for all $x, y, z \in X$.

(ii) $D^*(x, y, z) = 0$ if and only if $x = y = z$.

(iii) $D^*(x, y, z) = D^*(\sigma(x, y, z))$ for all $x, y, z \in X$

where $\sigma(x, y, z)$ is any permutation of the set $\{x, y, z\}$.

(iv) $D^*(x, y, z) \leq D^*(x, y, w) + D^*(w, z, z)$ for all $x, y, z, w \in X$.

The pair (X, D^*), where D^* is a generalized metric on X is called a **D*–metric space** or a **generalized metric space**.

Example 1.2: Let (X, d) be a metric space. Define $D_1^*: X^3 \rightarrow [0, \infty)$ by

$D_1^*(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$ for $x, y, z \in X$. Then (X, D_1^*) is a generalized metric space.

Example 1.3: Let (X, d) be a metric space. Define $D_2^*: X^3 \rightarrow [0, \infty)$ by

$D_2^*(x, y, z) = d(x, y) + d(y, z) + d(z, x)$ for $x, y, z \in X$. Then (X, D_2^*) is a generalized metric space.

Example 1.4: Let $X = R$, define $D^*: R^3 \rightarrow [0, \infty)$ by

$$D^*(x, y, z) = \begin{cases} 0 & if\ x = y = z \\ \max\{x, y, z\} & otherwise \end{cases}$$

Then (R, D^*) is a generalized metric space.

Note 1.5: Using the inequality in (iv) and (ii) of Definition 1.1, one can prove that if (X, D^*) is a D*–metric space, then

$D^*(x, x, y) = D^*(x, y, y)$ for all $x, y, \in X$.

Infact $D^*(x, x, y) \leq D^*(x, x, x) + D^*(x, y, y) = D^*(x, y, y)$ and

$D^*(y, y, x) \leq D^*(y, y, y) + D^*(y, x, x) = D^*(y, x, x)$, proving the inequity.

Definition1.6: Let (X, D*) be a D*-metric space. For $x \in X$ and $r > 0$, the set $B_{D^*}(x, r) = \{y \in X; D^*(x, y, y) < r\}$ is called the **open ball** of radius r about x.

For example, if X = R and $D^*: R^3 \rightarrow [0, \infty)$ is defined by

$D^*(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in R$. Then

$B_{D^*}(0, 1) = \{y \in R; D^*(0, y, y) < 1\}$

$= \{y \in R; 2|y| < 1\}$

$= \{y \in R; |y| < ½\} = (-½, ½)$.

Definition 1.7: Let (X, D^*) be a D*–metric space and $E \subset X$.

(i)If for every $x \in E$, there is a $\delta > 0$ such that $B_{D^*}(x, \delta) \subset E$, then E is said to be an **open subset** of X

(ii)If there is a $k > 0$ such that $D^*(x, y, y) < k$ for all $x, y \in E$ then E is said to be ***D*–* bounded.** It has been observed in [9] that, if τ is the set of all open sets in (X, D^*), then τ is a topology on X (called the **topology induced by the *D**–metric**) and also proved that $B_{D^*}(x, r)$ is an open set for each $x \in X$ and $r > 0$ ([9], Lemma 1.5). If (X, τ) is a compact topological space we shall call (X, D^*) is a **compact** D^*–metric space.

Definition 1.8: Let (X, D^*) be a D^*–metric space. A sequence $\{x_n\}$ in X is said to

(i) **converge to *x*** if $D^*(x_n, x_n, x) = D^*(x_n, x, x) \to 0$ as $n \to \infty$

(ii) be a **Cauchy sequence**, if to each $\in > 0$, there is a natural number n_0

such that $D^*(x_n, x_n, x_m) < \in$ for all $m, n \geq n_0$.

It is easy to see (infact proved in [9], Lemma 1.8 and Lemma 1.9) that, if $\{x_n\}$ converges to x in(X, D^*) then x is unique and that $\{x_n\}$ is a Cauchy sequence in (X, D^*). However, a Cauchy sequence in a (X, D^*) need not be convergent as shown in the example given below.

Example 1.9: Let X = (0, 1] and $D^*(x, y, z) = |x - y| + |y - z| + |z - x|$ for $x, y, z \in X$, so that (X, D^*) is a D^*–metric space.

Define $x_n = \frac{1}{n}$ for n = 1, 2, 3 ……., then $D^*(x_n, x_n, x_m) = 2|x_n - x_m| = 2\left|\frac{1}{n} - \frac{1}{m}\right|$, so that

$D^*(x_n, x_n, x_m) \to 0$ as $m, n \to \infty$, proving $\{x_n\}$ is a Cauchy sequence in (X, D*). Clearly $\{x_n\}$ does not converge to any point in X.

Definition 1.10: *A* D^*–metric space (X, D^*) is said to **complete** if every Cauchy sequence in it converges to some point in it.

It follows that the D^*–metric space given in Example 1.9 is not complete.

Note 1.11: We have seen (In Example 1.2 and Example 1.3) that on any metric space (X, *d), it* is possible to define at least two D^*–metrics, namely D_1^* and D_2^*, using the metric d. We shall call D_1^* and D_2^* as D^*–metrics induced by d. Thus every metric space (X, d) gives rise to at least two D^*–metric spaces (X, D_1^*) and (X, D_2^*). Also if (X, D^*) is a D^*–metric then defining $d_0(x, y) = D^*(x, y, y)$ for $x, y \in X$, we can show easily that (X, d_0) is a metric space and we shall call d_0 as a metric induced by D^*.

The following result is of use for our discussion.

Theorem 1.12: Let (X, d) be a metric space and D_i^* $(i = 1, 2)$ be the two D^*– metrics induced by d (given in Example 1.2 and Example 1.3). For any i (=1, 2) a sequence $\{x_n\}$ in $(X, D_i{}^*)$ is a Cauchy sequence if and only if $\{x_n\}$ is a Cauchy sequence in (X, d).

Proof: - First note that for i =1, 2 we have

$d(x, y) \le D_i{}^*(x, y, y) \le 2d(x, y)$ for all $x, y \in X$.

Now the theorem follows immediately in view of the above inequality.

For example, if $\{x_n\}$ is a Cauchy sequence in (X, d), then for any given $\epsilon > 0$ choose a natural number n_0 such that $m, n \ge n_0$ implies $d(x_m, x_n) < \frac{\epsilon}{2}$; and note that for the same n_0 we have $m, n \ge n_0$ implies $D_i{}^*(x_m, x_n, x_n) \le 2d(x_m x_n) < \epsilon$, proving that $\{x_n\}$ is a Cauchy sequence in $(X, D_i{}^*)$.

Similarly the other part of the theorem can be proved using the other inequality noted in the beginning of the proof.

Corollary 1.13: Suppose (X, d) is a metric space. Let $D_1{}^*$ and $D_2{}^*$ be two D^*– metrics induced by d, then for any i (=1, 2) the space $(X, D_i{}^*)$ is complete if and only if (X, d) is complete. **Proof: -** Follows from Theorem 1.12.

Definition 1.14: If (X, D*) is a D*-metric space, then D* is a **continuous function** on X^3, in the sense that $\lim_{n\to\infty} D^*(x_n, y_n, z_n) = D^*(x, y, z)$, whenever $\{(x_n, y_n, z_n)\}$ in X^3 converges to $(x, y, z) \in X^3$. Equivalently,

$$\lim_{n\to\infty} x_n = x,\ \lim_{n\to\infty} y_n = y,\ \lim_{n\to\infty} z_n = z \Leftrightarrow \lim_{n\to\infty} D^*(x_n, y_n, z_n) = D^*(x, y, z).$$

Notation: For any selfmap T of X, we denote T(x) by Tx.

If S and T are selfmaps of a set X, then any $z \in X$ *such that* $Sz = Tz = z$ is called a **common fixed point** of S and T.

Two selfmaps S and T of X are said to be **commutative** if ST = TS where ST is their composition SoT defined by (SoT) x = STx for all $x \in X$.

Definition 1.15: Suppose S and T are selfmaps of a D^*–metric space (X, D^*) satisfying the condition $T(X) \subseteq S(X)$. Then for any $x_0 \in X$, $Tx_0 \in T(X)$ and hence $Tx_0 \in S(X)$, so that there is a $x_1 \in X$ with $Tx_0 = Sx_1$, since $T(X) \subseteq S(X)$. Now $Tx_1 \in T(X)$ and hence there is a $x_2 \in X$ with $Tx_2 \in T(X) \subseteq S(X)$ so that $Tx_1 = Sx_2$. Again $Tx_2 \in T(X)$ and hence $Tx_2 \in S(X)$ with $Tx_2 = Sx_3$. Thus repeating this process to each $x_0 \in X$, we get a sequence $\{x_n\}$ in X such that $Tx_n = Sx_{n+1}$ for $n \ge 0$. We shall call this sequence as an **associated sequence of** x_0 **relative to the two selfmaps S and T.** It may be noted that there may be more than one associated sequence for a point $x_0 \in X$ relative to selfmaps S and T.

Let S and T are selfmaps of a D*-metric space (X, D*) such that T(X) ⊆ S(X). For any $x_o \in X$, if $\{x_n\}$ is a sequence in X such that $Tx_n = Sx_{n+1}$ for $n \geq 0$, then $\{x_n\}$ is called an ***associated sequence*** of x_0 relative to the two selfmaps S and T.

Definition 1.16 : A function Ø: [0,∞) → [0,∞) is said to be a ***contractive modulus***, if Ø(0) = 0 and Ø(t) < t for t > 0.

Definition 1.17: A real valued function Ø defined on X ⊆ R is said to be ***upper semi continuous***, if $\lim_{n\to\infty} \sup Ø(t_n) \leq Ø(t)$ for every sequence $\{t_n\}$ in X with $t_n \to t$ as $n \to \infty$.

Definition 1.18: If S and T are selfmaps of a D*-metric space (X, D*) such that for every sequence $\{x_n\}$ in X with $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$, we have

$\lim_{n\to\infty} D^*(STx_n, TSx_n, TSx_n) = 0$, then we say that S and T are ***compatible.***

2. THE MAIN RESULTS

2.1 Theorem. Suppose *S*, *T*, I and J be selfmaps of a *D**–metric space (*X*, *D**) satisfying the conditions

(i) S (X) ⊆ J (X) and T (X) ⊆ I (X)

(ii) D *(Sx, Ty, Ty) ≤ ρ (x, y) for all x, y ∈ X ,

where

(ii)′ ρ (x , y) = max{D *(Ix, Jy, Jy), D *(Ix, Sx, Sx), D *(Jy, Ty, Ty),

$$\frac{1}{2} D^*(Ix, Ty, Ty), \frac{1}{2} D^*(Jy, Sx, Sx)\} \text{ for } x, y \in X$$

(iii) *S, T*, I and J are continuous.

(iv) the pairs (*S* , *I*) and (T, J) are compatible,

and

(v) there is a point $x_0 \in X$ and an associated sequence $\{x_n\}$ of x_0 relative to the four selfmaps such that the sequences $\{Sx_{2n}\}$ and $\{Tx_{2n+1}\}$ converge to some point $z \in X$

Further, if

(vi) there exists $(a, b) \in X^2$ such that $f(a, b) = \sup_{(x,y)\in X^2} f(x,y)$, where

(vi)′ $f(x, y) = \frac{D*(Sx,\ Ty,\ Ty)}{\rho(x,y)}$

then S, T, I and J have a unique common fixed point z ϵ X. also z is the unique fixed point for the pair (S, I) and for the pair (T, J).

Proof: First suppose that ρ(x', y') = 0 for some x', y' ∈ X. Then

(2. 1. 1) max{D*(Ix', Jy', Jy'), D*(Ix', Sx', Sx'), D*(Jy', Ty', Ty'), $\frac{1}{2}$D*(Ix', Ty', Ty'), D*(Jy', Sx', Sx')} = 0,

which implies

(2.1.2) Ix' = Sx' = Jy' = Ty', and also

(2. 1. 3) SIx' = S(Sx') = S^2x' and

(2. 1. 4) TJy' = T(Ty') = T^2y'. Now since the pair (S, I) is compatible, we have

(2.1.5) $\lim_{n\to\infty} D^*$(SIy_n, ISy_n, ISy_n) = 0

whenever Sy_n,, Iy_n → t as n → ∞ for some t ∈ X. Letting y_n = x' for n ≥ 1, then Sy_n → Sx' and Iy_n → Ix' as n →∞. Therefore (2.1.5) gives that D*(SIx', ISx', ISx') = 0 or SI x' = IS x'. Also since ISx' = S^2x' = SIx' and Jy' = Ty' we get

ρ(Sx', y') = max {D*(ISx', Jy', Jy'), D*(ISx', S^2x', S^2x'), D*(Jy', Ty', Ty'), $\frac{1}{2}$ D*(IS x', Ty', Ty'), $\frac{1}{2}$ D*(Jy', S^2x', S^2x')}

= max {D*(S^2x', Ty', Ty'), 0, 0, $\frac{1}{2}$ D*(S^2x', Ty', Ty'), $\frac{1}{2}$ D*(S^2x', Ty', Ty')}

= D*(S^2x', Ty', Ty'). That is

(2. 1. 6) ρ(Sx′, y′) = D*(S^2x', Ty', Ty')

Now if Ty′ ≠ S^2x', then by (ii), we have

(2.1.7) D*(S^2x', Ty', Ty') < ρ(Sx', y')

Thus (2.1.6) and (2.1.7) contradict each other if Ty′ ≠ S^2x'. Therefore S^2x′ =Ty′. Further, from (2. 1. 2)

(2. 1. 8) S^2x′ = Ty' = S(Sx') =STy′ and so Ty' = z(say) is a fixed point of S. Again, by (2. 1. 2)

(2. 1. 9) Iz = ITy' = ISx' = STy' = Sz = z. Therefore Sz = Iz = z, showing that z is a common fixed point of S and I. Again since the pair (T, J) is compatible, we have

$\lim_{n\to\infty} D^*$(TJy_n, JTy_n, JTy_n) = 0. Whenever Ty_n,, Jy_n → t as n → ∞ for some t ∈ X. taking y_n = y', we find that Ty_n → Ty', Jy_n → Jy' as n →∞. Therefore (2.1.5) gives that

(2. 1. 10) D*(TJy', JTy', JTy') = 0 or TJy' = JTy'.

Now if Sx′ ≠ T^2y', then by (ii), we have

(2.1.11) D*(Sx', T^2y', T^2y') < ρ(x', Ty')

But, by (2. 1. 2) and (2. 1. 4)we have

ρ(x', Ty') = max {D*(Ix', JTy', JTy'), D*(Ix', Sx', Sx'), D*(JTy', T^2y', T^2y'), $\frac{1}{2}$ D*(Ix', T^2y', T^2y'), $\frac{1}{2}$ D*(JTy', Sx', Sx')}

$= D^*(Sx', T^2y', T^2y')$. That is,

(2. 1. 12) $\rho(x', Ty') = D^*(Sx', T^2y', T^2y')$

Thus (2. 1. 11) and (2. 1. 12) contradict each other if $Sx' \neq T^2y'$.

Therefore $Sx' = T^2y'$. hence, by (2. 1.10) and (2. 1. 2), we have

(2. 1. 13) $Sx' = T^2y' = T(Ty') = TJy' = JTy' = JSx'$, showing that $Sx' = z'$ is a fixed point of J. Further

(2. 1. 14) $Tz' = TSx' = TJy' = JTy' = JSx' = Jz' = z'$ and therefore $Tz' = Jz' = z'$, showing that z′ is a common fixed point of T and J.

Now we prove that $z = z'$.

First note that, if $z \neq z'$, then by (ii), we have

(2. 1. 15) $D^*(z, z', z') = D^*(Sz, Tz', Tz') < \rho(z, z')$. But

(2. 1. 16) $\rho(z,, z') = \max \{D^*(Iz, Jz', Jz'), D^*(Iz, Sz, Sz), D^*(Jz', Tz', Tz'), \frac{1}{2} D^*(I z, Tz', Tz'), \frac{1}{2} D^*(Jz',Sz,Sz)\} = 0$

$= \max \{D^*(z, z', z'), 0, 0, \frac{1}{2} D^*(z, z', z'), \frac{1}{2} D^*(z, z', z')\}$

$= D^*(z, z', z')$,

Since (2. 1. 15) and (2. 1. 16) contradict each other if $z \neq z'$, it follows that $z = z'$. Hence z is the unique common fixed point of S, T, I and J.

Now suppose that $\rho(x, y) > 0$ for all $x, y \in X$, so that f(x, y) is well defined. Now by the inequality (ii), we find that $f(x, y) < 1$ for all $x, y \in X$. Hence if $c = f(p, q)$ then $c \leq 1$, so that $f(x, y) \leq c$ for all $x, y \in X$ and therefore, from (vi)′ $D^*(Sx, Ty, Ty) \leq c\, \rho(x , y)$ for all $x, y \in X$

Since, by hypothese, all the conditions of the corollary holds for the four selfmaps S, T, I and J; it follows that they have a common fixed point $z \in X$. Further z is the unique common fixed point of S and I; and of T and J.

To prove the uniqueness of z, let w be another common fixed point of S, T, I and J.

If $w \neq z$, then by (ii), we have

(2. 1. 17) $D^*(z, w, w) = D^*(Sz, Tw, Tw) < \rho(z, w)$

(2. 1. 18) $\rho(z,, w) = \max \{D^*(Iz, Jw, Jw), D^*(Iz, Sz, Sz), D^*(Jw, Tw, Tw), \frac{1}{2} D^*(I z, Tw, Tw), \frac{1}{2} D^*(Jw, Sz, Sz)\}$

$= \max \{D^*(z, w, w), 0, 0, \frac{1}{2} D^*(z, w, w), \frac{1}{2} D^*(z, w, w)\}$

$= D^*(z, w, w)$,

Now (2, 1,17) and (2. 1. 18) contradict each other if $z \neq w$. Therefore $z = w$, showing z is the unique common fixed point of S, T, I and J. Further z is the unique common fixed point of S and I; and of T and J.

Now we prove some consequences of Theorem 2. 1

2.2 Corollary: Suppose (X, D*) is a D*-metric space and S, T, I and J are selfmaps of X satisfying conditions (i), (ii), (iii) and (iv) of Theorem 2.1. Further, if (X, D*) is compact, then S, T, I and J have a unique common fixed point z. Also z is the unique common fixed point for the pair S and I; and for the pair T and J.

Proof: Since (X, D*) is a compact D*-metric space, it is complete and therefore for each $x_0 \in X$ and for any associated sequence $\{x_n\}$ of x_0 relative to four selfmaps such that the sequences $\{Sx_{2n}\}$ and $\{Tx_{2n+1}\}$ converge to some $z \in X$ and hence condition (v) of Theorem 2.1 holds . Also, if (X, D*) is compact D*-metric space, then f(x, y) is a continuous function on the compact D*-metric space X^2. Therefore we can find $(a, b) \in X^2$ such that $f(a, b) = \sup_{(x,y)\in X^2} f(x, y)$, proving that the condition (vi) of the Theorem 2.1. Hence by Theorem 2. 1, the conclusion of the corollary follows.

2.3 Corollary ([5]): Suppose S, T, I and J are four selfmaps of metric space (X, d) such that

(i) $S(X) \subseteq J(X)$ and $T(X) \subseteq I(X)$

(ii) $d(Sx, Ty) < \rho_0(x, y)$ for all $x, y \in X$.

where

$$\rho_0(x, y) = \max \{d(Ix, Jy), d(Ix, Sx), d(Jy, Ty), \tfrac{1}{2} d(Ix, Ty), \tfrac{1}{2} d(Jy, Sx)\}$$

(iii) S, T, I and J are continuous on X. and

(iv) SI=IS and TJ = JT, further if

(v) X is compact.

Then the four selfmaps S, T, I and J have a unique common fixed point $z \in X$. Also z is the unique common fixed point of S and I; and of T and J.

Proof: Given (X, d) is a metric space satisfying condition (i) to (v) of the corollary.
If $D_1^*(x, y, z) = \max\{d(x, y), d(y, z), d(z, x)\}$, then (X, D_1^*) is a D*-metric space and $D_1^*(x, y, x) = d(x, y)$. Therefore (ii) can be written as $D^*(Sx, Ty, Ty) < \rho(x, y)$ for all $x, y \in X$, where $\rho(x, y) = \max \{D_1^*(Ix, Jy, Jy), D_1^*(Ix, Sx, Sx), D_1^*(Jy, Ty, Ty), \tfrac{1}{2} D_1^*(Ix, Ty, Ty), \tfrac{1}{2} D_1^*(Sy, Tx, Tx)\}$, which is the same as condition (ii) of Theorem 2.1. Also since (X, d) is complete, we have (X, D_1^*) is complete, by Corollary 1.13. Now S and T are selfmaps on (X, D_1^*) satisfying conditions of Corollary 2.2 and hence the corollary follows.

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